SHORT COMMUNICATION

An Improved Method for Computing Heat Accumulation from Daily Maximum and Minimum Temperatures

Naoshi Watanabe

Division of Entomology and Nematology, Yokohama Plant Protection Station, Yokohama 231, Japan

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Effective heat units or effective heat accumulation above a given threshold are conventionally estimated from daily mean temperature to predict the effect of temperature on biological processes. This, however, tends to underestimate the effective heat units when the daily maximum is higher and the mean is lower than the threshold. An improved method was proposed by Arnold (1960), taking both the daily maximum and minimum temperatures into account and applying the sine curve as a model for daily temperature change. Baskerville and Emin (1969) extended the Arnold’s method to introduce an upper threshold above which the biological process is inhibited completely or to a constant level. They compiled a table for calculating the heat units, in which the heat units above 51°C are read from the daily maximum and minimum temperatures. The heat units above a threshold other than 51°C can be obtained by shifting the temperature scale for reading the maximum and minimum temperatures. Their procedure, however, is somewhat troublesome and the table is not applicable to such a case as shown in Fig. 1, D. An attempt has been made to improve these points as described below.

Models for estimating the heat units in different scales and amplitude of temperature are shown in Fig. 1, in which the daily heat accumulation is represented by the shaded area. The symbols used in this figure are defined as follows.

\[ T_1 = \text{Daily minimum temperature.} \]
\[ T_2 = \text{Daily maximum temperature.} \]
\[ k_1 = \text{Lower threshold.} \]
\[ k_2 = \text{Upper threshold by which the heat accumulation is horizontally cut off.} \]
\[ m = \text{Daily mean temperature} = (T_1 + T_2)/2. \]
\[ R = \text{Difference between maximum and minimum temperature} = T_2 - T_1. \]

\[ \theta_1 = \sin^{-1} \frac{2(k_1 - m)}{R} \]
\[ \theta_2 = \sin^{-1} \frac{2(k_2 - m)}{R} \]
\[ \theta_3 = \sin^{-1} \frac{2(k_3 - m)}{R} \]

For convenience following parameters may be defined (see the figure at the bottom of Table 1).

\[ h = \frac{1}{\pi} \int_{\theta_1}^{\pi/2} \left( \frac{1}{2} \sin t - \frac{1}{2} + p \right) dt \] (1)
\[ q = \frac{1}{\pi} \left( \frac{\pi}{2} - \theta \right) \] (2)

where \[ \theta = \sin^{-1}(1 - 2p) \] (3)

**Case A:** Daily minimum below lower threshold \((k_1)\); daily maximum between lower threshold \((k_1)\) and upper one \((k_2)\).

This situation is shown in Fig. 1, A and equation giving the shaded area is defined as

\[ H = \frac{1}{\pi} \int_{\theta_1}^{\pi/2} \left( \frac{1}{2} \sin t + m - k_1 \right) dt \] (4)

where \(H\) represents the daily heat units.

The ratio of the difference between the maximum and lower threshold to the difference between the maximum and minimum temperatures is given by

\[ p_1 = \frac{T_2 - k_1}{R} \]

Replacing \(p\) by \(p_1\) gives \(h_1\) in eq. (3) and \(h_1\) in eq. (1), by which the heat units defined in eq. (4) can be simplified as

\[ H = h_1 R \] (5)

In order to facilitate this computation, Table 1 was compiled with an aid of FORTRAN program, in which \(h_1\) can be read from \(p_1\).

**Case B:** Daily minimum above lower threshold \((k_1)\); daily maximum below upper threshold \((k_2)\).

This situation is shown in Fig. 1, B and the heat units are calculated directly from the difference between the daily mean temperature and the lower threshold as

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Case C: Daily maximum between upper thresholds (k₂ and k₃); daily minimum below upper threshold (k₃).

This situation is shown in Fig. 1, C. The heat units are computed as follows: first, the total heat units for the day are computed without regard to the upper limit k₃; then, the heat units above the limit k₂ are obtained in the same manner as described in Case A. The heat units between the two limits k₁ and k₂ are given by the difference between the two values as

\[ H = H₁ - H₂ \]

where H₁ is given by eq. (5) when the minimum is below the limit k₁ or given by eq. (6) when the minimum is above the limit k₁, and H₂ is given as

\[ H₂ = \frac{1}{\pi} \int_{\theta₂}^{\pi/2} \frac{R}{2 \sin t + m - k₂} dt \]

The ratio of the difference between the maximum and the upper threshold (k₂) to the difference between the maximum and minimum is given as

\[ p₂ = \frac{T₂ - k₂}{R} \]

Replacing p by p₂ gives \( q₂ \) in eq. (3) and \( h₃ \) in eq. (1), by which eq. (7) can be simplified as

\[ H₃ = h₃R \]

The value of \( h₃ \) can also be read from \( p₃ \) in Table 1.

Case D: Daily maximum above upper threshold (k₂); daily minimum below upper threshold (k₃).

This situation is shown in Fig. 1, D. The heat units are computed as follows: first, the heat units without regard to the upper limit k₃ are computed in the same manner as described in Case C; then, the heat units between the two limits k₁ and k₂ are eliminated when the temperature exceeds the upper threshold (k₃). This procedure is expressed as

\[ H = H₁ - H₂ - H₃ \]

where \( H₁ \) and \( H₂ \) can be obtained by the procedure as described above and \( H₃ \) is defined as below.

\[ H₃ = (k₂ - k₁)q₃ \]

where \( q₃ \) is the daily duration of time above the threshold temperature (k₃) and expressed as

\[ q₃ = \frac{1}{\pi} \left( \frac{\pi}{2} - \theta₃ \right) \]

The ratio of the difference between the maximum and upper threshold (k₂) to the difference between the maximum and minimum is given as

\[ p₃ = \frac{T₂ - k₃}{R} \]
Table 1. Parameters for Rapid Calculation of Effective Heat Units

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Replacing $p$ by $p_3$ gives $\theta_s$ in eq. (3), by which $q_3$ can be obtained in eq. (2). The value of $q_3$ can also be read from $p_3$ in Table 1.

**Case E:** Both the daily minimum and maximum between upper thresholds ($k_3$ and $k_4$).

This situation is shown in Fig. 1, E and equation for the computation is expressed simply as

$$H = k_3 - k_1$$

**Case F:** Daily maximum above upper threshold ($k_3$); daily minimum between upper thresholds ($k_2$ and $k_3$).

This situation is shown in Fig. 1, F and the heat units are given as

$$H = (k_2 - k_1)(1 - q_3)$$

where $q_3$ can be obtained by the procedures as mentioned in **Case D**.

Thus the computation of daily heat accumulation may be greatly improved by using the table of parameters. Furthermore, the duration of time above the threshold temperature on each day may also be inferred from the table, which may contribute ecological studies in other aspects.

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**REFERENCES**
